Short-time dynamics of a two-dimensional majority vote model

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Short-time Monte Carlo methods are used to study the nonequilibrium ferromagnetic phase transition in a majority vote model in two dimensions. The existence of an initial critical slip regime is verified. The measured values of dynamic exponents $z=2.170(5)$ and $\theta=0.191(2)$ are in excellent agreement with those of the kinetic Ising model universality class. [S1063-651X(98)00301-8]

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The identification and characterization of universality classes for nonequilibrium systems are far less settled than in the case of systems in thermal equilibrium. Nevertheless, many model systems with microscopic irreversible dynamic rules (no detailed balance) and two states per site have been found to fall in the Ising model universality class as far as the static behavior is concerned [1–8]. Grinstein, Jayaprakash, and Yu He [9] have argued that, provided the rules are up-down symmetric, both the statics and the (long-time) dynamics of those models are the same as those of the kinetic Ising model. However, we are not aware of any direct determination of the dynamic z exponent for such models. In the present work we have applied the recently proposed early-time dynamic Monte Carlo technique [10–12] to investigate the dynamic behavior of a model in the above-mentioned class: the majority vote model (whose rates can be seen as a combination of two Glauber dynamics in contact with two heat baths at different temperatures) [13,1]. Previous studies of short-time dynamics were concerned either with equilibrium systems, such as Ising [14,11,12] or Potts [15–17] models, or with a nonequilibrium phase transition in a distinct universality class [18].

Janssen, Schaub, and Schmittmann [10] have shown that when a system with relaxational dynamics is quenched from $T \gg T_c$ to T_c, the early times of evolution also display universal behavior. An independent exponent θ, associated with the anomalous dimension of the initial order parameter, was introduced to describe the system behavior during this *critical initial slip* regime. Denoting by m_0 the initial magnetization ($0<m_0 \ll 1$), this regime is found in the time range $t_{mic}<t<m_0^{-z/x_0}$, where t_{mic} is some microscopic time and $x_0=\theta z+\beta/\nu$ (β and ν are the equilibrium critical indices). The magnetization $[m(t)=N^{-1}\Sigma_i\langle\sigma_i\rangle]$ increases with time as

$$m(t)\sim m_0 t^{\theta}. \tag{1}$$

θ is also related to the decay of the autocorrelation function from a disordered initial state

$$A(t)\sim t^{-\lambda}, \tag{2}$$

with $\lambda=d/z-\theta$ in d space dimensions. The relation between short-time dynamics and damage spreading was recently clarified by Grassberger [19].

For a finite system it is expected [10,11] that the moments of the order parameter $m^{(k)}$ (kth moment of the magnetization) have the scaling form

$$m^{(k)}(t,\tau,L,m_0)=b^{-k\beta/\nu}m^{(k)}(b^{-z}t,b^{-1/\nu}\tau,bL,b^{-x_0}m_0), \tag{3}$$

where $\tau=(T-T_c)/T_c$ and the initial correlation length is null. Setting $\tau=0$ and the arbitrary scaling factor $b\sim t^{1/z}$, one obtains from Eq. (3)

$$m(t,L,m_0)\sim t^{-\beta/\nu z}F(t/t_L,t/t_0), \tag{4}$$

where $t_L\sim L^z$ and $t_0=m_0^{-z/x_0}$. Following the scaling relations for the magnetization and its higher moments, it is possible to infer that the time-dependent Binder cumulant [20]

$$U(t,L)=1-\frac{m^{(4)}}{3(m^{(2)})^2} \tag{5}$$

obeys

$$U(t,L_1)=U(b^{-z}t,L_2) \tag{6}$$

for $\tau=0$, $m_0=0$, and two system sizes (L_1 and L_2) with $b=L_2/L_1$. The exponent z can be obtained from a data collapse with a time rescaling factor b^{-z}. Since only early times are considered, this is a rather efficient method to extract z. Once z is known, the static exponent β/ν is recovered from a similar scaling analysis of $m^{(2)}$.

Starting with random initial configurations (with $m_0=0$) and following the evolution at T_c of the spin autocorrelation function

$$A(t)=\frac{1}{N}\left\langle\sum_{i=1}^{N}\sigma_i(0)\sigma_i(t)\right\rangle, \tag{7}$$

the power-law decay (2) is visible after a short transient regime and λ (and therefore θ) can be obtained [16]. A direct measurement of θ is possible from Eq. (1): The samples are then prepared with a sharply defined small value of m_0. After a few Monte Carlo steps (MCS) a straight line appears in the log-log plots and θ is computed from its slope.

The two-state isotropic majority vote model is defined [1] by a set of "voters" or "spin" variables $\{\sigma_i\}$ taking the

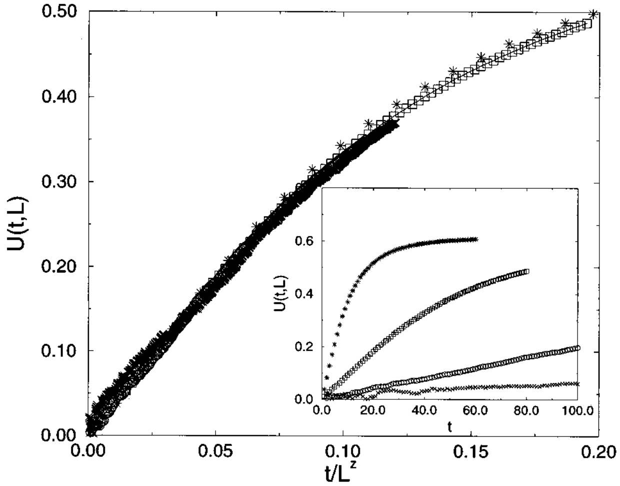

FIG. 1. Collapse plot of the Binder cumulant $U(t,L)$ as a function of t/L^z for $L=8$ (*), 16 (□), 32 (○), and 64 (×) with $z=2.170$. The inset shows $U(t,L)$ against t (same symbols).

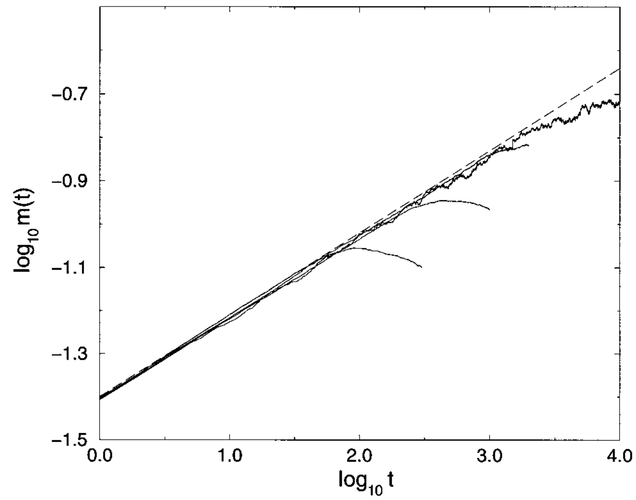

FIG. 2. A log-log plot of the time evolution of the magnetizatiom $m(t)$ for system sizes $L=16$, 32, 64, and 128 from bottom to top. The initial magnetization was $m_0=0.031\,25$. The dashed guide line has slope 0.191 and was plotted for comparison.

values $+1$ or -1 and evolving in time by a single spin-flip-like dynamics with a probability W_i given by

$$W_i(\sigma)=\frac{1}{2}\left[1-\sigma_i(1-2q)S\left(\sum_{\delta}\ \sigma_{i+\delta}\right)\right], \tag{8}$$

where $S(x)=\mathrm{sgn}(x)$ if $x=0$, $S(x)=0$ if $x=0$, and the sum is over nearest neighbors of σ_i. The control parameter q plays the role of temperature in equilibrium systems and measures the probability of aligning antiparallel to the majority of neighbors. In two dimensions this model has a ferromagnetic stationary phase for $0\leqslant q\leqslant q_c$ undergoing a second-order phase transition to a paramagnetic phase at q_c [$q_c=0.075(1)$ for a square lattice [1,5]]. The static critical behavior is Onsager-like [1,5,8]. According to the argument of Grinstein *et al.* [9], its dynamic critical behavior is the same as model A (Ising) [21] and therefore the renormalization-group analysis of Janssen *et al.* should also apply to such a nonequilibrium model. The aim of this paper is to report a direct confirmation of the above conjecture from the results of a short-time dynamics study of this model. The exponents z and θ are found to be indistinguishable from the corresponding Ising values, $z=2.172(6)$ [19], $z=2.1665(12)$ [22], and $\theta=0.191(3)$ [19] and the existence of an intermediate scaling regime is verified.

Simulations were carried out for square lattices of side $L=16$, 32, 64, and 128 with periodic boundary conditions. Random initial configurations with $m_0=0$ were used in the study of $U(t)$ and $A(t)$, whereas a small excess of plus spins, randomly distributed on the lattice, was taken to produce a selected value of $m_0=0$. In order to prepare a sample with a precise magnetization and negligible correlation length, we generated a lattice state with equal probability of occupation for both spin states and then flipped the spin at randomly chosen sites until the desired magnetization was obtained. The lattice was updated by flipping randomly picked spins [23] with probability given by Eq. (8) with $q=q_c=0.075$. The evolution was followed for up to 1000 MCS. Averages were performed over a large number of histories (up to 4×10^5 independent initial configurations).

In Fig. 1 Binder's cumulant $U(t,L)$ is displayed against t/L^z for different values of system size and initial magnetization $m_0=0$. The value of the z exponent obtained from the best collapse was $2.170(5)$, which is in very good agreement with the ones obtained by Grassberger [19] from a damage spreading study and by Nightingale and Blöte [22] from a variance-reducing Monte Carlo algorithm for the two-dimensional Ising model with nonconservative dynamics. Figure 2 shows a plot of $\log m(t)$ against $\log t$ for $m_0=0.031\,25$ and different system sizes ($L=16$, 32, 64, and 128); for comparison a straight line with slope 0.191 is also drawn. The dependence of θ on m_0 was analyzed (see Table I). A linear extrapolation to $m_0=0$ yields $\theta=0.191(2)$. It is clear from Fig. 2 that θ can be estimated from the study of very small system sizes.

In our case, even for $L=16$, the power-law behavior lasts for two decades of MCS with an exponent close to our best value. This shows that for the measurement of θ the finite-size effects are not important, in contrast with the behavior of the autocorrelation function, where higher values of L are necessary. From Fig. 2 we can also obtain an estimate of the crossover time t_c when the magnetization changes to the decreasing power-law behavior ($t^{-\beta/\nu z}$) before entering the ultimate exponential regime. It is clear that for $t<t_c$ the magnetization presents a power-law increase for all values of L with no significative finite-size effects. It is also remark-

TABLE I. Exponent θ as a function of m_0 for $L=32$. The value obtained for $m_0\to0$ is $\theta=0.191(2)$, in good agreement with the values in the literature [16,19].

m_0	θ
0.023 437 500	0.1870(20)
0.031 250 000	0.1856(20)
0.058 593 750	0.1809(20)
0.080 078 125	0.1783(20)
$m_0\to0$	0.191(2)

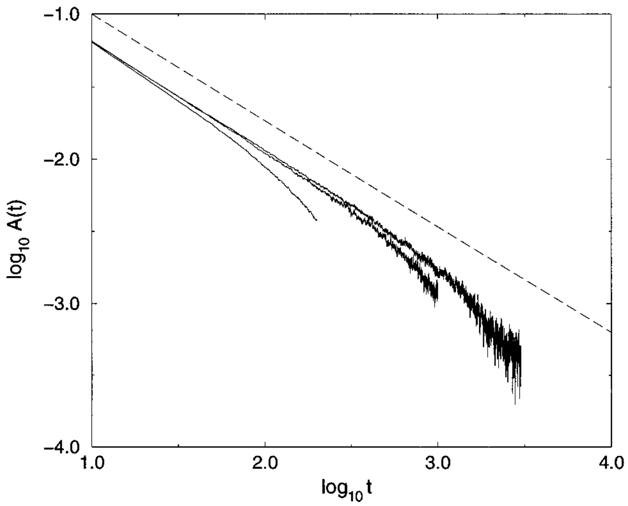

FIG. 3. A log-log plot of the autocorrelation function $A(t)$ for system sizes $L=32$, 64, and 128 (bottom to top) with initial magnetization $m_0=0$. The dashed guide line has slope 0.735.

able that this power law appears at a very small time ($t_{mic}\sim 1$ MCS).

In Fig. 3 we show a double-logarithmic plot of the autocorrelation function $A(t)$ as a function of time for different values of L. To get the critical exponent λ we discarded the first 10 MCS.

The best value obtained for λ was $0.735(5)$, which agrees with recently published results [16]. The scaling relation $\lambda=d/z-\theta$ is well obeyed for the values obtained before, within the statistical errors.

In summary, we have investigated the dynamic behavior of a critical nonequilibrium model, the two-dimensional majority vote model, making use of the early-time dynamic Monte Carlo method. By following the time evolution of the magnetization, Binder cumulant, and time autocorrelation function for systems of various sizes, we were able to calculate numerically the values of the exponents $z=2.170(5)$, $\theta=0.191(2)$, and $\lambda=0.735(5)$. These values are in very good agreement with their corresponding two dimensional Ising results: a direct confirmation of the stability of the kinetic Ising fixed point with respect to irreversibility of the microscopic rates (of a certain kind). The effect of the absence of detailed balance probably has to be sought in properties such as the cluster structure and the dynamics of pattern formation [24]. Another dynamic exponent θ_1, the global persistent exponent, was recently introduced by Majumdar *et al.* [25]. It measures the persistence of the sign of the magnetization and is related to θ for a Markovian system [25]. Evidence of non-Markovian nature was reported for a nonequilibrium model [26], but the situation is unclear in the Ising case [25,27]. The persistence probability of the majority vote model is currently being investigated.

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